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**LETTER TO THE EDITOR**

# Collective excitations of a single skyrmion in two dimensions at high magnetic field

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**Abstract.** The propagation of spin waves in the presence of a single skyrmion was considered. It was found that short-wavelength spin waves are hardly scattered, and that the main effect consists in an additional rotation of the spin deviation around the unperturbed spin direction in skyrmions. A special branch of collective excitations was found corresponding to oscillations of the skyrmion core size.

Since the pioneering theoretical paper [1], there have been a large number of articles published concerning a special kind of distortion of the uniform ferromagnetic state for odd-Landau-level fillings of 2D electron systems. These distortions, called skyrmions, are characterized by the rotation of the average spin vector on a large scale defined by Zeeman and Coulomb energies with nontrivial topological properties. It was shown that the creation of two such excitations with opposite charges (and topological numbers) has lower energy than that of a spin exciton [2, 3] at small enough $g$-factors. The question of the energy of one isolated skyrmion was not addressed. In later work [4, 5], a kind of gradient expansion was developed to calculate the skyrmion energy. The authors of [4, 5] assumed that the electron spinors belong to the same Landau-level states before and after the transformation induced by application of the nonuniform rotation matrix $U(\boldsymbol{r})$. This reduced rotation matrix was not unitary: $U^+(\boldsymbol{r})U(\boldsymbol{r}) \neq 1$. The consideration in [4, 5] is invalid for the isolated skyrmion also.

These inconsistencies can be removed [6–8] by using a subsequent transformation induced by application of a nonreduced rotation matrix and considering the full Schrödinger equation obtained by means of ordinary perturbation theory applied to the gradients of this matrix. The various physical quantities were calculated in the first and zero order of the expansion in inverse electron mass (or in cyclotron frequency). The results show that the formation of the isolated skyrmion gives rise to an additional ‘effective’ spin-dependent vector potential. If the corresponding total ‘effective’ magnetic field is lower than the external one, the creation of the isolated skyrmion with the appropriate topological number gives the gain in thermodynamic energy. If the degree of mapping is zero (two skyrmions with opposite topological numbers), this gain will be absent and the results of [1] are valid. The topological Hopf term in the skyrmion action was found. Its value corresponds to Fermi statistics in the sense of reference [9].

It is possible to use here the technique developed for the investigation of collective modes of single skyrmions. Collective modes of skyrmion systems (skyrmion crystals) were considered in [10] and will not be discussed here. We shall use the general semiclassical method to solve the linearized classical equation of motion and carry out its subsequent quantization. This method is used for example in the theory of spin waves [11]. In [8] the same method was applied to achieve the quantization of the skyrmion motion as a whole. It was shown that a skyrmion neutralized by an additional bound electron has a lower energy than a charged one due to its lack of oscillation in the external magnetic field.

I will now give a general outline of the theory. I give first a short description of the procedure followed to obtain the action depending on the skyrmion rotation matrix $U(\boldsymbol{r})$. Details can be found in [7]. First, one must carry out the unitary transformation of the electron spinors $\psi(\boldsymbol{r}) = U(\boldsymbol{r})\chi(\boldsymbol{r})$ to new spinors $\chi$. The electron Lagrangian transforms from

$$L_0 = \int \Bigg\{ \mathrm{i}\psi^+ \frac{\partial \psi}{\partial t} - \psi^+ \left( -\mathrm{i}\,\frac{\partial}{\partial \boldsymbol{r}} + A_0 \right)^2 \psi - \frac{1}{2} \int V(\boldsymbol{r}-\boldsymbol{r}')\psi^+(\boldsymbol{r})\psi^+(\boldsymbol{r}')\psi(\boldsymbol{r}')\psi(\boldsymbol{r})\ \mathrm{d}r' \Bigg\}\ \mathrm{d}r\ \mathrm{d}t \tag{1}$$

to

$$L = \int \Bigg\{ \mathrm{i}\chi^+ \frac{\partial \chi}{\partial t} - \chi^+ \left( -\mathrm{i}\,\frac{\partial}{\partial \boldsymbol{r}} + \boldsymbol{A}_0 - \mathrm{i}U^+ \frac{\partial U}{\partial \boldsymbol{r}} \right)^2 \chi + \mathrm{i}\chi^+ U^+ \frac{\partial U}{\partial t}\chi - \frac{1}{2} \int V(\boldsymbol{r}-\boldsymbol{r}')\chi^+(\boldsymbol{r})\chi^+(\boldsymbol{r}')\chi(\boldsymbol{r}')\chi(\boldsymbol{r})\ \mathrm{d}r' \Bigg\}\ \mathrm{d}r\ \mathrm{d}t. \tag{2}$$

We use units in which $\hbar = 1$, the magnetic length squared $l_H^2 = c\hbar/(eB) = 1$, and the external magnetic field $B = 1$ (normal to the 2D plane). $\boldsymbol{A}_0$ is the appropriate vector potential. The rotation matrix can be defined by three Euler angles [12]:

$$U(r) = U_z(\gamma)U_y(\beta)U_z(\alpha)$$

where the subscripts denote the appropriate axes of the rotation, and the $z$-axis coincides with the direction of the magnetic field. It is supposed that after the transformation the correct solution of the Schrödinger equation will be close to the solution in which the spinor $\boldsymbol{\chi}$ has only one component $\chi$ with $\sigma_z\chi \approx \chi$. Untransformed spinors $\psi$ are obtained by rotation of this spinor $\chi$ by any form of $U$. In order to have a nonzero degree of mapping of the 2D plane onto a sphere, which is required by definition for a skyrmion, one must have polar-angle singularity in the angles $\alpha$ and $\delta$ for our choice of $z$-axis. The transformed Lagrangian contains the quantities $-\mathrm{i}U^+\,\partial_\nu U = \Omega_\nu^l \sigma_l$, where $\nu = (x, y, z)$ and the $\sigma_l$ are the Pauli matrices, with

$$\begin{aligned} \Omega_\nu^z &= \frac{1}{2}(\partial_\nu \alpha + \sin\beta\ \partial_{\nu\gamma}) \\ \Omega_\nu^x &= \frac{1}{2}(\sin\beta\cos\alpha\ \partial_\nu\gamma - \sin\alpha\ \partial_\nu\beta) \\ \Omega_\nu^y &= \frac{1}{2}(\sin\beta\sin\alpha\ \partial_\nu\gamma + \cos\alpha\ \partial_\nu\beta). \end{aligned} \tag{3}$$

The transformation performed will be nonsingular only if the singularities of $\alpha$ and $\gamma$ are the same and occur at the point where $\cos\beta = -1$. Furthermore, we consider the simplest case with unit degree of mapping and symmetrical $\alpha = \gamma = \varphi$, where $\varphi$ is a polar angle defined with respect to the skyrmion centre at which $\cos\beta(\boldsymbol{r}_0) = -1$. The degree of mapping is given by the integral

$$\frac{1}{2\pi} \int \mathrm{rot}\,\boldsymbol{\Omega}^z\ \mathrm{d}^2 r = Q$$

where $Q$ is an arbitrary integer.

Now it is possible to proceed in the usual Hartree–Fock way by dividing the Hamiltonian into two parts:

$$H_0 = \int \left\{ \frac{1}{2m\chi} \chi^+ \left( -\mathrm{i}\,\frac{\partial}{\partial \boldsymbol{r}} + \boldsymbol{A}_0 + \boldsymbol{\Omega}^l \sigma_l \right)^2 \chi + \chi^+ \Omega_t^l \sigma_l - \int V(\boldsymbol{r} - \boldsymbol{r}') \langle \chi_\lambda^+(\boldsymbol{r}')\chi_\mu(\boldsymbol{r})\rangle \chi_\mu^+(\boldsymbol{r})\chi_\lambda(\boldsymbol{r}')\ \mathrm{d}^2 r' \right\} \mathrm{d}^2 r\ \mathrm{d}t \tag{4}$$

and the interaction part

$$H_{int} = -\frac{1}{2} \int V(\boldsymbol{r} - \boldsymbol{r}')\chi_\lambda^+(\boldsymbol{r})\chi_\mu(\boldsymbol{r}')[\chi_\mu^+(\boldsymbol{r}')\chi_\lambda(\boldsymbol{r}) - 2\langle \chi_\mu^+(\boldsymbol{r}')\chi_\lambda(\boldsymbol{r})\rangle]\ \mathrm{d}^2 r\ \mathrm{d}^2 r'. \tag{5}$$

Here, we denote the uniform average at $H_0$ with $\Omega_\nu^l = 0$ by the angle brackets; the subscript attached to $\chi$ denotes the corresponding spin component.

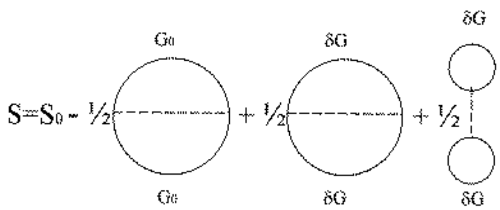


**Figure 1.** A graphical representation of the skyrmion action, Solid lines correspond to the parts of the electron Green function identified in the picture. Broken lines correspond to the interaction potential. The quantity $S_0$ corresponds to the action with the Hamiltonian $H_0$, including all of the corrections due to the nonuniformity of the rotation matrix.

The corresponding action depending on the matrix $U$ can be obtained within the Hartree–Fock approximation as shown in figure 1. Here the first term corresponds to the electron Green function with the Hamiltonian (4). $G = G_0 + \delta G$ where $G_0$ is the reduced form with $\Omega_\nu^l = 0$. The interaction potential is denoted by broken lines. We neglect here the Zeeman term, assuming a small value of the $g$-factor, and consider it later in the first approximation of the perturbation theory.

The Green function $G_0$ has the form

$$G_0(\boldsymbol{r}, \boldsymbol{r}', t - t') = \sum_{s,p} \int g_s(\omega) \mathrm{e}^{\mathrm{i}\omega(t'-t)} \Phi_{sp}(\boldsymbol{r}) \Phi_{sp}^+(\boldsymbol{r}')\ \frac{\mathrm{d}\omega}{2\pi} \tag{6}$$

where

$$g_0 = \frac{1}{\omega + (J - \mathrm{i}\delta)\sigma_z}$$

$$g_{s\neq 0} = \frac{1}{\omega + J\sigma_z + \mathrm{i}\delta} \qquad \delta \to +0$$

and $\Phi_{sp}$ are normalized wave functions of Landau level $s$. We assume that only the lowest level, $s = 0$, with spin up is fully filled and that the others are empty. The quantity

$$J = \frac{e^2}{2l_H}\sqrt{2\pi}$$

is the exchange energy per electron in the uniform ferromagnetic state. In such a way, the skyrmionic action can be obtained from a gradient expansion in terms of $\Omega_\nu^l$.

The thermodynamic energy $\langle H - \mu N \rangle$, where $\mu$ is the chemical potential and $N$ is the number of particles, was obtained in [7, 8]; it contains some topologically invariant terms, the most important of which is proportional to $\omega_c$ and gives the gain in the thermodynamic energy for skyrmions with negative $Q$. These terms do not contribute to the equations of motion and are not essential for the consideration of collective modes. At small distances from the skyrmion centre the gradient energy prevails; it can be expressed either in terms of $\Omega^l$ or directly in terms of the Euler angles $\alpha, \beta$:

$$E_0 = \frac{1}{2} J' \int \left( \frac{\partial n_j}{\partial x_l} \right)^2 \mathrm{d}^2 r \tag{7}$$

with

$$J' = \frac{1}{16\sqrt{2\pi}} \frac{e^2}{l_H}$$

and

$$\boldsymbol{n} = (\sin\beta\cos\alpha, \sin\beta\sin\alpha, \cos\beta).$$

$E_0$ is not topologically invariant but its minimal value is proportional to $|Q|$ and does not depend on the skyrmion size $L_c$ [13]. At distances of the order of $L_c$ and beyond, one needs to take into account the next small terms: the Zeeman energy

$$E_Z = \int g\mu_B H (n_z - 1) \, \frac{\mathrm{d}^2 r}{2\pi}$$

and the Coulomb energy

$$E_C = \frac{e^2}{l_H} \frac{A}{L}$$

with the constant $A$ depending on the charge distribution. According to the minimal solution [13],

$$n_z - 1 \sim \left( \frac{L_c}{r} \right)^2 \left( -\frac{1}{2} \right)$$

and we get for the size-dependent energy, with logarithmic accuracy,

$$E_1 = -g\mu_B H \frac{L_c^2}{2\pi} \ln \frac{L^*}{L_c} + \frac{Ae^2}{l_H L_c} \tag{8}$$

where $L^*$ is the distance at which the gradient energy is of the order of the Zeeman energy:

$$L^* = \sqrt{\frac{J'}{g\mu_B H}}$$

and the Zeeman energy decreases exponentially. The core size is defined by the minimum of equation (8), and we have, with logarithmic accuracy,

$$L_c^3 \ln \frac{L^*}{L_c} = \frac{Ae^2}{g\mu_B H l_H}.$$

It is essential that at distances $L \sim L_c$ the gradient energy is still large compared to the Zeeman and Coulomb energy, assuming a small value of the $g$-factor. At larger distances the orientation of the average spin is close to the direction of the magnetic field, and all situations can be treated by means of perturbation theory.

Along the lines of the above consideration, I will now investigate the collective modes—spin waves in the exchange approximation—but neglecting both the Zeeman and the Coulomb energies. The variation of the skyrmion action

$$S_{ex} = -\int \left[ \Omega_t^z + \frac{1}{2} J' \left( \frac{\partial n_j}{\partial r_k} \right)^2 \right] \mathrm{d}^2 r \; \mathrm{d}t \tag{9}$$

in $\alpha, \beta$ gives the classical Landau–Lifshitz equation

$$\boldsymbol{n} \times \frac{\partial \boldsymbol{n}}{\partial t} = J' \, \boldsymbol{\nabla n}$$

which, after using the linearization $\boldsymbol{n} = \boldsymbol{n}_0(\boldsymbol{r}) + \boldsymbol{m}(\boldsymbol{r}, t)$, acquires the form

$$\boldsymbol{n}_0 \times \frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{\nabla m} \tag{10}$$

where $\boldsymbol{n}_0(\boldsymbol{r})$ is the unperturbed spin direction in the skyrmion.

The complexity of this equation is connected with the change of $\boldsymbol{n}_0$ in the 2D plane. Introducing a rotating coordinate system, we assume that $\boldsymbol{n}_0 = V \check{\boldsymbol{z}}, \boldsymbol{m} = V \boldsymbol{m}'$ where $V(\boldsymbol{r}) = V_z(\alpha) V_y(\beta) V_z(\gamma)$ is the rotation matrix for the average spin direction expressed in terms of the same three Euler angles, $\alpha, \beta$ and $\gamma = \alpha = \varphi$, with $\check{\boldsymbol{z}}$ denoting the unit vector in the $z$-direction. The matrices $V$ are no longer $2 \times 2$ spinor matrices; they are now $3 \times 3$ rotation matrices that can operate on three-component vectors.

After carrying out this rotation, we get

$$\check{\boldsymbol{z}} \times \frac{\partial \boldsymbol{m}'}{\partial t} = J' V^+ \, \boldsymbol{\nabla} (V \boldsymbol{m}').$$

It is easy to show using the identity $V^+ V = 1$ that the last equation can be recast in the form

$$\check{\boldsymbol{z}} \times \frac{\partial \boldsymbol{m}'}{\partial t} = J' \left( \frac{\partial}{\partial \boldsymbol{r}} + V^+ \frac{\partial V}{\partial \boldsymbol{r}} \right)^2 \boldsymbol{m}'. \tag{11}$$

This is analogous to the tranformation from expression (1) to expression (2). By direct differentiation, one can show that

$$V^+ \frac{\partial}{\partial \boldsymbol{r}} V = \Gamma^l \tau_l$$

where

$$\begin{aligned} \Gamma^1 &= (1 + \cos\beta) \boldsymbol{\nabla} \varphi \\ \Gamma^2 &= (\sin\varphi \, \boldsymbol{\nabla} \beta - \sin\beta \cos\varphi \, \boldsymbol{\nabla} \varphi) \\ \Gamma^3 &= (\cos\varphi \, \boldsymbol{\nabla} \beta + \sin\beta \sin\varphi \, \boldsymbol{\nabla} \varphi) \end{aligned} \tag{12}$$

and the antisymmetric matrices $\tau_l$ given by

$$\tau_1 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{13}$$

$$\tau_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \tag{14}$$

$$\tau_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \tag{15}$$

correspond to infinitesimal rotations.

The solution of equation (11) can be obtained by means of perturbation theory applied to the $\Gamma^l$ and their derivatives. We consider here only the matrix $\Gamma^1\tau_1$ corresponding to the rotation of the vector $\boldsymbol{m}'$ around the direction of $\check{\boldsymbol{z}}$. To take into account the other matrices, one needs to carry out the calculation of the component $m'_z$ which corresponds to higher orders in the gradient expansion.

Introducing polar coordinates with respect to the skyrmion centre, it is easy to transform equation (11) using expressions (12)–(15) to the following form:

$$-\mathrm{i}\frac{\omega}{J'}m'_y = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}m'_x + \left(\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{(1+\cos\beta)^2}{r^2}\right)m'_x - \frac{2(1+\cos\beta)}{r^2}\frac{\partial}{\partial\varphi}m'_y$$

$$\mathrm{i}\frac{\omega}{J'}m'_x = \frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}m'_y + \left(\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + \frac{(1+\cos\beta)^2}{r^2}\right)m'_y + \frac{2(1+\cos\beta)}{r^2}\frac{\partial}{\partial\varphi}m'_x.$$

Here $\omega$ is the frequency of the spin wave. The latter equations can be written in compact Schrödinger-like forms in terms of the complex function $\psi = m'_x + \mathrm{i}m'_y$. Assuming that $\psi = \mathrm{e}^{\mathrm{i}n\varphi}R_n(r)$, we obtain

$$\frac{\omega}{J'}R_n = \left[-\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{(n+1+\cos\beta)^2}{r^2}\right]R_n. \tag{16}$$

We consider here only spin waves with large wave vectors:

$$k^2 = \frac{\omega}{J'} \qquad kL_c \gg 1.$$

The opposite case, $kL_c \ll 1$, can be considered also, but is of less interest due to its small statistical weight and will not be discussed here.

The function $\beta = \beta(r/L_c)$ is slowly varying with the radii, and the solution of (16) will be close to free-wave solution $R_n = J_n(kr) + \delta R_n$ where $J_n(kr)$ is a Bessel function. The correction $\delta R_n$ can be obtained in the first order of perturbation theory:

$$\delta R_n = J_n(kr)\int_\infty^r J_n(kr')Y_n(kr')F(r')\,\mathrm{d}r' + Y_n(kr)\int_0^r J_n^2(kr')F(r')\,\mathrm{d}r'. \tag{17}$$

Here $Y_n(kr)$ is the other (singular) solution of the Bessel equation, and

$$F(r) = \frac{2n(1+\cos\beta)}{r}. \tag{18}$$

The scattering amplitude $f_n \sim 1/(kL_c)$ is given at the second term in (17) and can be calculated using the asymptotes of the Bessel functions for $kr \gg 1$. The scattering at large angles is defined by $n \sim 1$ and the cross section for large-angle scattering $\sigma \sim (1/(kL_c))^2$ is small. The scattering at small angles is not so significant physically. The main effect of the skyrmion consists in the additional rotation of the spin-deviation vector $\boldsymbol{m} = V\boldsymbol{m}_0$ where $\boldsymbol{m}_0 = (\mathrm{Re}\,\psi, \mathrm{Im}\,\psi, 0)$ and $\psi = m_0\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$. In one sense the behaviour of the spin waves in the presence of a skyrmion is an analogue of the Bohm–Aharonov effect, in which the vector potential only changes the phase of the wave function outside the solenoid with integer numbers of flux quanta.

I now consider the special dilatation mode. In the above, I did not consider the collective mode in which only the Euler angle $\beta$ depends on time while the other the angles are unchanged. Such oscillation changes the skyrmion size, and the main gradient energy is excluded because it does not depend on the core size. In such a case, the term containing the time derivative in the action (9) vanishes also, and one needs to find the next term in the expansion in time derivatives.

For the calculation, we shall assume that $\beta = \beta_0(r/L(t))$ where $\beta_0$ is the static solution for the skyrmion [13]. In Lagrangian (2), the corresponding time derivative generates the term

$$-H_1 = \mathrm{i}\int \chi^+ U^+ \frac{\partial U}{\partial t}\chi \ \mathrm{d}^2 r = \sum_{l\neq z}\int \chi^+ \boldsymbol{\Omega}^l \boldsymbol{r}\sigma_l \chi \ \mathrm{d}^2 r \ \left(\frac{1}{L}\frac{\partial L}{\partial t}\right). \tag{19}$$

Because $\boldsymbol{r}\cdot\nabla\varphi = 0$, the term containing $\boldsymbol{\Omega}^z$ vanishes, and

$$\boldsymbol{r}\boldsymbol{\Omega}^x = \frac{1}{2}r\frac{\partial\beta}{\partial r}\sin\varphi \qquad \boldsymbol{r}\boldsymbol{\Omega}^y = \frac{1}{2}r\frac{\partial\beta}{\partial r}\cos\varphi.$$

The first-order term in the skyrmion action vanishes:

$$S_1 = \mathrm{Tr}\int H_1 G_0 \ \mathrm{d}^2 r \ \mathrm{d}t = 0$$

($G_0$ is given by (6)). It can be shown that the mixed term containing $\partial L/\partial t$ and the space derivatives of $\boldsymbol{\Omega}^l$ vanishes also. One needs to find the second-order term in $\partial L/\partial t$. Using standard perturbation theory for the action, we get

$$S_2 = \frac{\mathrm{i}}{2}\mathrm{Tr}\int H_1 G_0 H_1 G_0 \ \mathrm{d}^2 r \ \mathrm{d}t \tag{20}$$

where we omit for brevity intermediate integrations over space and time. In the calculation of (20) one must take into account only the terms with $s = 0$ in Green function (6), because other terms give small contributions of the order of $m \sim 1/\omega_c$:

$$S_2 = \frac{\mathrm{i}}{2}\mathrm{Tr}\int H_1(\boldsymbol{r},t) g_0(\omega)\Phi_{0p}(\boldsymbol{r})\Phi^+_{0p}(\boldsymbol{r}') \times H_1(\boldsymbol{r},t) g_0(\omega)\Phi_{0p'}(\boldsymbol{r}')\Phi^+_{0p'}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega\delta} \ \frac{\mathrm{d}\omega}{2\pi} \ \mathrm{d}^2 r' \ \mathrm{d}^2 r' \ \mathrm{d}t$$

we take into account only $(\partial L/\partial t)^2$, neglecting higher-order terms in the time derivatives and space derivatives of $\boldsymbol{\Omega}^l$, due to the nonlocality. The contribution to the action gives only cross terms containing the singularities in $g_0$ above and below the real $\omega$-axis. Using the orthogonality properties of $\Phi_{sp}$ and performing the summation over $p$, one has

$$S_2 = \frac{1}{8J}\frac{\partial^2 E_1}{\partial L^2}\int \left(r\frac{\partial\beta}{\partial r}\right)^2 \ \frac{\mathrm{d}^2 r}{2\pi} \ \left(\frac{1}{L}\frac{\partial L}{\partial t}\right)^2 \ \mathrm{d}t. \tag{21}$$

The space integral diverges logarithmically and we get, with logarithmic accuracy, using the linearization $L = L_c + \delta L$,

$$S_2 = \frac{1}{8J}\left(\ln\frac{L^*}{L_c}\right)\int\left(\frac{\partial\,\delta L}{\partial t}\right)^2 \ \mathrm{d}t. \tag{22}$$

We must calculate also the change in the skyrmion energy due to the change of the skyrmion size using the expression (8) for the Zeeman and Coulomb energies, omitting the gradient energy which does not depend on it:

$$\delta E = \frac{1}{2}\frac{\partial^2 E_1}{\partial L^2} = -\frac{3}{2}g\mu_B H\left(\ln\frac{L^*}{L_c}\right)(\delta L)^2.$$

Therefore we have an ordinary oscillator total action:

$$S_2 = \int\left[\frac{1}{8J}\left(\ln\frac{L^*}{L_c}\right)\left(\frac{\partial\,\delta L}{\partial t}\right)^2 + \frac{3}{2}g\mu_B H\left(\ln\frac{L^*}{L_c}\right)(\delta L)^2\right] \ \mathrm{d}^2 r \ \mathrm{d}t$$

with the oscillator frequency

$$\hbar\omega = \sqrt{12 g \mu_B H J}$$

where $J$ is the exchange energy per electron in the uniform ferromagnetic state. The oscillator frequency is consequently smaller than

$$J = \frac{e^2}{2l_H}\frac{1}{\hbar}\sqrt{2\pi}$$

due to the assumed small value of the $g$-factor. The result obtained is valid with logarithmic accuracy.

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